WHEN IS A FIXED NUMBER OF OBSERVATIONS OPTIMAL?

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1. Introduction

Many of the classical fixed sample size tests and estimates have sequential counterparts which are more economical, needing on the average fewer observations to ensure a given performance. It turns out, however, that under some circumstances, admittedly artificial, a sample of fixed, nonrandom size is optimal.

We determine here rather inclusive conditions ensuring that for a sequence of partial sums of independent, identically distributed random variables, a fixed sample size is optimal with respect to a given nonnegative payoff function.

2. Notations

Let $X_1, X_2, \cdots$ be independent and identically distributed replicates of a random variable X, and set $S_0 = 0$, $S_n = X_1 + X_2 + \cdots + X_n$, $n \geqq 1$.

Let M be the set of all real numbers α for which $\varphi(\alpha)$, the moment generating function of X, is finite: $\varphi(\alpha) = E(\exp\{\alpha X\})$ and $M = \{\alpha | \varphi(\alpha) < \infty\}$. The set M is an interval containing $\alpha = 0$, and may consist of all the real numbers, a subinterval of them, or the sole value zero.

The nonnegative function $r_n(x)$, $n = 0, 1, \cdots$, x real, will be called the payoff function in the sense that if one stops observations after n trials his income is $r_n(S_n)$.

The optimal stopping problem is to determine a stopping time N, if possible, such that

$$E(r_N(S_N)) = \sup_T E(r_T(S_T)), \tag{1}$$

where the sup on the right is taken over all stopping times T. When such a stopping time N exists, we denote its "value" by V; that is, V is the maximal expected payoff given by (1); $V = E(r_N(S_N))$.

The pair (n, x) is called *accessible* if S_n is contained in every neighborhood of x with positive probability. Clearly, the value of $r_n(x)$ at inaccessible points is irrelevant.

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