NECESSARY CONDITIONS FOR DISCRETE PARAMETER STOCHASTIC OPTIMIZATION PROBLEMS

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1. Introduction

Consider the following formal optimization problem. Let $\{\xi_i\}$ denote a sequence of random vectors, and define the sequence (1.1) of n dimensional vectors $\{X_i, i = 0, \cdots, k\}$, $X_i = \{X_i^1, \cdots, X_i^n\}$, where k is a fixed integer and u_i is a control, which is an element of an abstract set $\tilde{U}_i$:

$$X_{i+1} = X_i + f_i(X_i, u_i, \xi_i). \tag{1.1}$$

The object is to find the $\{u_i\}$ which minimizes

$$\begin{aligned} EX_k^0 &\equiv \sum_{i=0}^{k-1} f_i^0(X_i, u_i, \xi_i), \\ X_{i+1}^0 &= X_i^0 + f_i^0(X_i, u_i, \xi_i), \qquad X_i^0 \text{ fixed}, \end{aligned} \tag{1.2}$$

subject to certain constraints. Sometimes it is convenient to augment the vector X_i by adding X_i^0, the "cost" component. Then, we write ${}^+\underline{X}_i = (X_i^0, X_i)$, $\underline{f}_i = (f_i, f_i^0)$ and

$$\underline{X}_{i+1} = \underline{X}_i + \underline{f}_i(X_i, u_i, \xi_i). \tag{1.1'}$$

The constraints are

$$r_0(X_0) \equiv E\tilde{r}_0(X_0) = 0, \qquad q_0(\underline{X}_0) \equiv E\tilde{q}_0(\underline{X}_0, E\underline{X}_0) \leqq 0, \tag{1.3}$$

$$\begin{aligned} q_i(X_k) &\equiv E\tilde{q}_i(X_i, EX_i) \leqq 0, \qquad i = 1, \cdots, k, \\ r_k(X_k) &\equiv E\tilde{r}_k(X_k, EX_k) = 0, \end{aligned} \tag{1.4}$$

where $\tilde{r}_0$, $\tilde{q}_0$, $\tilde{r}_k$, and $\tilde{q}_i$ are vector valued functions. The q_0 is allowed to depend on X_0^0 in order to fix or limit X_0^0 in some way. That is, some component of $\tilde{q}_0(\underline{X}_0)$ may be $\tilde{q}_0^0(\underline{X}_0) = -X_0^0 \leqq 0$.

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