# ASYMPTOTIC DISTRIBUTION OF EIGENVALUES OF RANDOM MATRICES

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## 1. Introduction

The impetus for this paper comes mainly from work done in recent years by a number of physicists on a statistical theory of spectra. The book by M. L. Mehta [10] and the collection of reprints edited by C. E. Porter [14] are excellent references for this work. The discussion in Section 1.1 is an attempt to present a rationale for such investigations. Our interpretation of linear operators as used in quantum mechanics is based largely on the book by T. F. Jordan [8].

1.1. *Statistical theory of spectra.* In quantum mechanics knowledge of the value of measurable quantities of a system is expressed in terms of probabilities. A state of the system specifies these probabilities. Measurable quantities are represented by self-adjoint linear operators on a separable Hilbert space. The only possible values of the measurable quantities are those in the spectrum of the self-adjoint operator which represents the measurable quantity.

Experience indicates that energy is represented by the Hamiltonian operator. We are interested in the point spectrum of the Hamiltonian, which is its set of eigenvalues. The eigenvalues $E$ of the Hamiltonian operator $H$, which are real since $H$ is self-adjoint, are those values of energy for which some state of the system specifies a probability of one that the energy is exactly equal to $E$ [8]. This is expressed in the Schrödinger time independent equation,

$$H\psi = E\psi, \tag{1.1.1}$$

where $\psi$ is an eigenvector associated with $E$.

In ordinary statistical mechanics, renunciation of exact knowledge of the state of a system is made and only properties of averages are considered. An exact knowledge of the laws governing the system is assumed known; it is the impossibility in practice of observing the state of the system in all its detail that leads to the consideration of properties of averages.

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