PRESSURE AND HELMHOLTZ FREE ENERGY IN A DYNAMIC MODEL OF A LATTICE GAS

RICHARD HOLLEY
PRINCETON UNIVERSITY

1. Introduction

In this paper, we will study a model of an infinite volume one dimensional lattice gas. Our model differs from the usual model of a lattice gas in that the configuration of particles is a stochastic process. That is, the particles in the system will move around, and we will be studying properties of the system which are related to the motion. The particular interaction which governs the behavior of each particle will be introduced in Section 2. This interaction was discovered by F. Spitzer [4].

In Section 2, we will define the Helmholtz free energy in the usual way and prove that at constant temperature the Helmholtz free energy does not increase with time. In thermodynamics, this is usually derived as a consequence of the second law of thermodynamics. In Section 3, we will use the results obtained in Section 2 to prove that all shift invariant equilibrium states are limiting Gibbs distributions. Finally, in Section 4, we use the intuitive description of the interaction of the particles to motivate a definition for the pressure of a state. The usual definition of pressure used in statistical mechanics is only given for limiting Gibbs distributions, and the two definitions do not agree there. However, we will show that when they are both defined, they are both strictly increasing functions of the particle density at constant temperature. In the case of the usual definition this is well known.

In order to keep the notation as simple as possible, we will only consider one model in this paper. This model can clearly be generalized in several ways. Many of these generalizations can be found in [4]. The techniques in this paper are adequate to handle some, though by no means all, of these generalizations.

2. Helmholtz free energy

2.1. *Intuitive description.* We begin by giving an intuitive description of the stochastic process. For a careful proof that this process really exists the reader is referred to [1].

This work was prepared while the author was a Miller Fellow in the Statistics Department, University of California, Berkeley.