COVERAGE OF GENERALIZED CHESS BOARDS BY RANDOMLY PLACED ROOKS

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1. Introduction

At a recent colloquium on combinatorial structures, H. Kamps and J. van Lint presented a paper [2] on the minimal number of rooks $\sigma(n, k)$ required to "cover" a generalized chessboard; the latter is represented by R_k^n, the set of n vectors (or cells) with components in the ring of integers mod k. To explain the notion of "cover" we first define the Hamming distance $d_H(\mathbf{x}, \mathbf{y})$ between two vectors ("squares" of the chessboard) as the number of components in which they differ; under the metric d_H, the board R_k^n is a metric space. The familiar chessboard is R_8^2. Then the rook domain or region *covered* by a rook at x is the unit sphere

$$B(x, 1) = \{y \in R_k^n | d_H(\mathbf{x}, \mathbf{y}) \leqq 1\}. \tag{1.1}$$

Kamps and van Lint gave the following table of $\sigma(n, k)$ which represents almost all the known results to date for the above deterministic problem.

TABLE I

KNOWN VALUES OF $\sigma(n, k)$

k \ n	3	4	5	6	7	8	⋯	13
2	2	4	7	12	16	2^5		
3	5	9	3^3					3^{10}
4	8	24	4^3					
5	13			5^4				
6	18	72						
7	25					7^6		

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