FIRST EMPTINESS PROBLEMS IN QUEUEING, STORAGE, AND TRAFFIC THEORY

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1. Introduction

One of the aims of the Berkeley Symposium is to encourage research workers to present a summary of results newly obtained in their fields during the previous five years. In accordance with this intention, the earlier part of the present paper will describe some interesting developments in problems of first emptiness since 1965. For simplicity, only first passage problems (to the zero state) for certain discrete time random walks on the integers $0, 1, 2, \cdots$, will be discussed. As is already known, first emptiness probabilities are of considerable importance in queueing, storage, and traffic problems. Their distributions may be interpreted

(a) in *queueing theory*, as probability distributions of the length of a busy period during which all waiting customers have been served, so that the queue is empty;

(b) in *storage theory*, as probability distributions of the times to first emptiness of a reservoir, all the stored water having been released;

(c) in *traffic theory*, as probability distributions of periods to a first gap at a "give way" intersection on a minor road, all vehicles crossing the road having passed, so that the intersection becomes empty and through traffic on the road can proceed.

A graphical representation of a random walk in discrete time of the type which arises in queueing, storage, and traffic processes is provided in Figure 1. Here $Z_0 = u$ is the initial state of the random walk at time $t = 0$; this represents the number of customers initially waiting for service in a queue, the units of water initially contained in a reservoir, or the number of vehicles initially waiting to cross a minor road, thus blocking the traffic along it.

The sequence of discrete nonnegative random variables $\{X_t\}_{t=0}^{\infty}$ constitutes the inputs into the system during the time intervals $(t, t + 1)$, $t = 0, 1, \cdots$. At the end of each time interval, there is a unit output if the random walk lies in any one of the states $1, 2, 3, \cdots$, or a zero output if it is in state zero. Inputs represent new arrivals at a queue, new water inflows into a reservoir, or new

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