USES OF THE SOJOURN TIME SERIES FOR MARKOVIAN BIRTH PROCESS

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1. Introduction

This paper will be concerned with the Markovian birth process, and in this section we shall establish notation and mention some properties of the process. We suppose that a sequence $\{\lambda_j : j = 1, 2, \cdots\}$ of positive constants is given. Development of the process Z_t is controlled by the conditions

$$P\{Z_{t+\delta t} = k \mid Z_t = j\} = \begin{cases} \lambda_j \delta t + o(\delta t) & \text{when} \quad k = j + 1, \\ 1 - \lambda_j \delta t + o(\delta t) & \text{when} \quad k = j, \\ o(\delta t) & \text{when} \quad k \neq j + 1, j. \end{cases} \tag{1.1}$$

We suppose that $Z_0 = 1$. In view of well-known applications of this model, it is sometimes convenient to refer to Z_t as the population size.

Let T_n be the epoch of the nth jump in the process Z_t for $n = 1, 2, \cdots$, and write $T_0 = 0$. Let X_n be the sojourn time in state n, that is to say, $X_n = T_n - T_{n-1}$. A well-known property of the process is that the X_n are independent and that

$$P\{X_j \leqq x\} = 1 - e^{-\lambda_j x}. \tag{1.2}$$

The mean and variance of the jth sojourn time are

$$EX_j = \lambda_j^{-1}, \qquad \text{Var } X_j = \lambda_j^{-2}, \tag{1.3}$$

respectively. In this paper, we shall make use of the random series formed by the sojourn times when centered at their means. The nth partial sum S_n of this series is given by

$$S_n = \sum_{j=1}^{n} (X_j - EX_j) = T_n - ET_n. \tag{1.4}$$

An important property of the birth process is whether or not it is "honest," that is, whether or not

$$\sum_{n=1}^{\infty} P\{Z_t = n\} = 1 \qquad \text{for all} \quad t \geqq 0. \tag{1.5}$$

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