A METHOD FOR STUDYING THE INTEGRAL FUNCTIONALS OF STOCHASTIC PROCESSES WITH APPLICATIONS, III

PREM S. PURI
PURDUE UNIVERSITY

1. Introduction

This paper is a continuation of the results presented in two earlier papers [20], [21] and may be read as the sequel. A brief account of their results will, however, be given here in order to make this paper selfcontained. The subject under study is the distribution of the integrals of the form

$$Y(t) = \int_0^t f(X(\tau), \tau)\, d\tau, \tag{1.1}$$

where $X(t)$, $t \geqq 0$, is a continuous time parameter stochastic process defined on a probability space $(\Omega, \mathscr{A}, \mathscr{P})$, with $\mathscr{X}$ as its state space, and f is a nonnegative (measurable) function defined on $\mathscr{X} \times [0, \infty)$. Here it is assumed that the integral $Y(t)$ exists and is finite almost surely for every $t > 0$.

The integrals $Y(t)$ arise in several domains of application such as in the theory of inventories and storage (see Moran [13], Naddor [14]), and in the study of the cost of the flow stopping incident involved in the automobile traffic jams (see Gaver [9], Daley [4], and Daley and Jacbos [5]). Such integrals are also encountered in certain stochastic models suitable for the study of response time distributions arising in various live situations (see Puri [16], [18], [19]). In fact in [18], it was shown that such a distribution is equivalent to the study of an integral of the type (1.1).

In [20], the work done by several authors in the past on the integral functionals of stochastic processes was briefly surveyed. But more importantly a method was introduced for obtaining the distribution of $Y(t)$. This method is based on a "quantal response process" $Z(t)$ defined for a hypothetical animal. By definition $Z(t)$ equals one if the animal is alive at time t and is equal to zero otherwise. In particular, it is assumed that

$$P(Z(t + \Delta t = 0 | Z(t) = 1, X(t) = x) = \delta f(x, t)\Delta t + o(\Delta t), \tag{1.2}$$

This research was supported in part by research grants NONR 1100(26) and N00014-67-A-0226-0008, from the Office of Naval Research.