THE DISTRIBUTION OF GENERATIONS AND OTHER ASPECTS OF THE FAMILY STRUCTURE OF BRANCHING PROCESSES

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1. Introduction

Throughout this paper branching processes will be viewed as models for the development of biological populations. Unless stated otherwise, it will be assumed that the population starts at time $t = 0$ with one individual in generation 0 and of age 0. Each member of the population will live for a random life time. Then he will be replaced by a random number of new individuals, his sons. These will be in generation $k + 1$ if their father was a member of the kth generation. Also we shall allow for an individual to "survive," that is, he may have himself as one offspring in generation k and start a new life. In the branching process model, it is further assumed that the lifetimes of all individuals have a common probability distribution with distribution function G, that the probability β for survival is the same for all individuals, that all individuals have the same distribution of the number of offspring given by a probability generating function h, and further, that all the random variables introduced so far are independent.

We shall follow the notations of Harris [10] who has studied many aspects of this model. Until recently the emphasis has been on studying the total population size; the possibility of an individual giving birth to offspring more than once has not usually been considered. The only exception seems to be the papers by Crump and Mode [5], [6], who consider a case somewhat more general than ours.

The questions with which this paper is concerned are about the distribution of generations in a population at a given time, the time pattern according to which generations appear and disappear, the degree of relationship between different individuals, the number of relatives of a certain degree, and so forth.

The first mention of distribution of generations with the present meaning in the literature was by Harris [10] who used the number $Z^{(k)}(t)$ of individuals in generations $0, 1, \cdots, k - 1$ alive at time t as an approximation to the total

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