ON BASIC RESULTS OF POINT PROCESS THEORY

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1. Introduction

There are many existing approaches to the theory of point processes. Some of these—following the original work of Khinchin [9] are "analytical" and others (for example, [15], [8]) quite abstract in nature. Here we will take a position somewhat in the middle in describing the development of some of the basic theory of point processes in a relatively general setting, but by using largely the simple techniques of proof described for the real line in [11]. We shall survey a number of known results—giving simple derivations of certain existing theorems (or their adaptations in our setting) and obtain some results which we believe to be new. Our framework for describing a general point process will be essentially that of Belyayev [2], while that for Section 4 concerning Palm distributions is developed from the approach of Matthes [14].

First we give the necessary background and notation. There are various essentially equivalent ways of defining the basic structure of a point process. For example, for point processes on the line, one may consider the space of integer valued functions $x(t)$ with $x(0) = 0$, which increase by a finite number of jumps in any finite interval. The events of the process then correspond to jumps of $x(t)$. One advantage of such a specification is that multiple events fit naturally into the framework.

To define point processes on an arbitrary space T, it is often appropriate to consider the "sample points" ω to be subsets of T. This is the point of view taken in [18], where each ω is itself a countable subset of the real line, the set of points "where events occur." Sometimes, however, a point process arises from some existing probabilistic situation (such as the zeros of a continuous parameter stochastic process) and one may wish to preserve the existing framework in the discussion. A convenient structure for this is the following, used in [2]. Let $(\Omega, \mathscr{F}, P)$ be a probability space and $(T, \mathscr{T})$ a measurable space (T is the space "in which the events will occur"). For each $\omega \in \Omega$, let S_ω be a subset of T. If for each $E \in \mathscr{T}$

$$N(E) = N_\omega(E) = \text{card}\,(E \cap S_\omega) \tag{1.1}$$

is a (possibly infinite valued) random variable, then S_ω is called a *random set* and the family $\{N(E): E \in \mathscr{T}\}$ a *point process*. The "events" of the process are, of course, the points of S_ω.