

MULTIVARIATE POINT PROCESSES

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1. Introduction

We consider in this paper events of two or more types occurring in a one dimensional continuum, usually time. The classification of the events may be by a qualitative variable attached to each event, or by their arising in a common time scale but in different physical locations. Such multivariate point processes, or multitype series of events, are to be distinguished from events of one type occurring in an n dimensional continuum and considered, for example, by Bartlett [2]. It is of course possible to have multivariate point processes in, say, two dimensions, for example, the locations of accidents labelled by day of occurrence, but we do not consider this extension here.

Multivariate series of events arise in many contexts; the following are a few examples.

EXAMPLE 1.1. Queues are a well-known situation in which bivariate point processes arise as the input and output, although interest in the joint properties of the input and output processes is fairly recent (for example, Daley [16] and Brown [7]). The two processes occur simultaneously in time. Many of the variants on simple queueing situations which have been considered give rise to more than two point processes.

EXAMPLE 1.2. An important and rich source of multivariate point processes is neurophysiology (Perkel, Gerstein, and Moore [41]). Information is carried along nerve bundles by spikes which occur randomly in time. (The spikes are extremely narrow and, at least in many situations, their shape and height do not appear to vary or carry information.) The neuronal spike trains of different types may be observations at different locations with no definite knowledge of physical connection, or may be the inputs and outputs to nerve connections (neurons).

EXAMPLE 1.3. When the events are crossings of a given level by a real valued stochastic process in continuous time, the up crossings and down crossings of the

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