NONHOMOGENEOUS POISSON FIELDS OF RANDOM LINES WITH APPLICATIONS TO TRAFFIC FLOW

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1. Introduction

This study was prompted by investigations of models of traffic flow on a highway through analyses of the structure and properties of Poisson fields of random lines in a plane. It is possible to view the trajectory of a car produced by its time and space coordinates on the highway as a straight line in that plane if the car travels at a constant speed once it enters the highway and then never leaves the highway. These traffic considerations plus the property of time invariance for traffic flow distributions lead to one model for traffic flow on a divided highway developed by Rényi [10]. This idealized model is simpler to study than the more realistic situation that provided Rényi's motivation and which he also subjects to analysis, namely, cars do lose time because of an overtaking of one car by another even on a divided highway with two lanes for traffic moving in one direction.

In his paper, Rényi found it convenient to start from the stochastic process of entrance times of the cars at a fixed point on the highway. Other authors start from the spatial process of cars distributed in locations along the highway at some fixed time according to some random law. The traffic flow results of Weiss and Herman [13] who study the spatial process for the idealized model are analogous to Rényi's results which stem from the temporal process. To demonstrate the equivalence of the two results, care must be taken to employ the appropriate measure in deriving distributions related to traffic flow. Both Rényi, and Weiss and Herman, achieved asymptotic results for traffic flow distributions. We will reproduce both results in Section 4 as special cases of our development of traffic flow models through the structure of random lines in the plane. It should be mentioned here that Brown [3] reconsiders Rényi's idealized model and derives exact distributions rather than asymptotic distributions for spatial and speed distributions of cars.

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