RANDOM FIELDS OF SEGMENTS AND RANDOM MOSAICS ON A PLANE

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1. Introduction

The position of an undirected segment of straight line of length τ in a Euclidean plane is determined by the triple coordinate $X = (x, y, \varphi)$, where x and y are the cartesian coordinates of the center of the segment and φ is the angle made by the segment with the zero direction. Let $\mathscr{X}$ denote the phase space of segment coordinates, that is, the layer in the three dimensional Euclidean space defined by the inequalities $-\infty < x < \infty$, $-\infty < y < \infty$, $0 < \varphi < \pi$.

Let $\mathscr{X}$ be a subset of the phase space $\mathscr{X}$ and let τ be a positive real function defined for $X \in \mathscr{X}$. Assign a length $\tau(X)$ to the segment which occupies the position X. This defines a certain set J of segments in the plane. We shall write $J = [\mathscr{X}; \tau(X)]$.

Call **I** the set of all those J for which the number of segments which intersects every bounded subset of the plane is finite. Moreover, if $J \in \mathbf{I}$, then, by definition, any two segments of J either do not intersect or they intersect at a single point.

Take a Borel set B in the phase space and a Borel set $T \subset (0, \infty)$. Each such pair (B, T) defines a subset of **I**, namely, the set of those $J = [\mathscr{X}; \tau(X)]$ such that $\mathscr{X} \cap B$ contains exactly one point and such that $X_0 \in \mathscr{X} \cap B$ implies $\tau(X_0) \in T$. Also, for each B, consider the subset of **I** formed by those J such that $\mathscr{X} \cap B = \varnothing$. The sets just introduced will be called cylindrical subsets of **I**.

Let $\mathscr{B}$ denote the minimal σ-algebra generated by the cylindrical subsets of **I** and let $(\Omega, \mathscr{A}, \mu)$ be a probability space.

DEFINITION 1. *A* $(\mathscr{B}, \mathscr{A})$ *measurable map* $\omega \rightsquigarrow J(\omega)\Omega$ *of* Ω *into* **I** *is called a random field of segments* (*r.f.s.*) *in the plane.*

If an r.f.s. $J(\omega)$ is given, then a probability measure P will be induced in $\mathscr{B}$, which we shall call the distribution of the r.f.s. $J(\omega)$.

The group of all Euclidean motions of a plane induces a group of transformations of $\mathscr{B}$ into itself (the group of motions of $\mathscr{B}$). An r.f.s. is called homogeneous and isotropic (h.i.r.f.s.), if its distribution is invariant with respect to the group of motions of $\mathscr{B}$. Only homogeneous and isotropic random fields of segments are examined herein.