BIRTH AND DEATH OF MARKOV PROCESSES

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1. Introduction

We must start with some basic notation and definitions. To avoid stopping the process at time 0, we shall deal with one particular situation, leaving it to the specialist to check whether our conclusions remain true when all hypotheses are deleted. Let E be a locally compact space with countable base. Some $\Delta \in E$ has been singled out for infamous purposes. Let Ω be the set of all mappings $\omega : \mathbf{R}_+ \rightsquigarrow E$ which are right continuous and possess a "lifetime" ζ (possibly 0 or $+\infty$), namely,

$$(1.1) \qquad \omega(t) \neq \Delta \text{ for } t < \zeta(\omega), \qquad \omega(t) = \Delta \text{ for } t > \zeta(\omega).$$

We set as usual $X_t(\omega) = \omega(t)$, $X_\infty(\omega) = \Delta$, and provide Ω with the natural family of σ-fields $(\mathscr{F}_t^0)_{t \leqq \infty}$ of the process (X_t). Given now a Hunt transition semigroup $(P_t)_{t \geqq 0}$ on E, with Δ as an absorbing point, we can define as usual measures P^μ, P^x on Ω, for which the process (X_t) is Markovian, with the transition semigroup (P_t) and initial measures μ, ε_x. The assumptions concerning left limits in the definition of Hunt processes will be superfluous most of the time. We postpone all other definitions to the main text.

We are interested in operations on the sample paths which preserve the homogeneous Markov character of the process, with possible alteration of the semigroup. Known examples of these operations are: turning a set into an absorbing barrier; restarting the process at a stopping time (we shall use the terminology "optional r.v." rather than "stopping time"); killing the process at a terminal time; reversing time at an L time (L times are called cooptional random variables below); clock changing relative to a continuous additive functional. Our purpose here consists in giving two more examples of such transformations.

Let us say informally that a positive random variable R is a *birth time* for the process if the process $(X_{R+t})_{t \geqq 0}$ starting at time R is, for every law P^μ, a homogeneous Markov process (its transition semigroup may depend on R, but not on μ). Similarly, replacing the process starting at R by the process killed at R, we