STOCHASTIC DIFFERENTIAL EQUATIONS AND MODELS OF RANDOM PROCESSES

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1. Description of a desirable model

Let us suppose that we are investigating a system whose state can be adequately specified by n real numbers $x^1, \cdots, x^n$. We shall suppose that by some acceptable scientific theory it is predicted that, in the absence of disturbances from outside the system, the x^i develop in time in accordance with certain differential equations,

$$\dot{x}^i = g_0^i(t, x), \qquad i = 1, \cdots, n. \tag{1.1}$$

If there are disturbances or noises, $n^1(t), \cdots, n^r(t)$, the underlying theory of such systems will often permit us to conclude that

$$\dot{x}^i = g_0^i(t, x) + \sum_{\rho=1}^{r} g_\rho^i(t, x) n^\rho(t), \qquad i = 1, \cdots, n, \tag{1.2}$$

where g_ρ^i is the sensitivity of the ith coordinate to the ρth noise. However in the underlying theory, equation (1.2) will usually have a limited domain of applicability; in particular, we could not usually retain confidence in the trustworthiness of (1.2) if the noise were unbounded. But for sufficiently well-behaved bounded noises we can rewrite (1.2) in the form

$$dx^i = g_0^i(t, x)\, dt + \sum_\rho g_\rho^i(t, x)\, dz^\rho, \tag{1.3}$$

or

$$x^i(t) = x_0^i + \int_a^t g_0^i[s, x(s)]\, ds + \sum_\rho \int_a^t g_\rho^i[s, x(s)]\, dz^\rho(s), \tag{1.4}$$

where

$$z^\rho(t) = z^\rho(a) + \int_a^t n^\rho(s)\, ds; \tag{1.5}$$

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