POISSON POINT PROCESSES ATTACHED TO MARKOV PROCESSES

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1. Introduction

The notion of point processes with values in a general space was formulated by K. Matthes [4]. A point process is called *Poisson*, if it is σ-discrete and is a renewal process. We will prove in this paper that such a process can be characterized by a measure on the space of values, called the *characteristic measure*.

Let X be a standard Markov process with the state space S. Fix a state $a \in S$ and suppose that a is recurrent state for X. Let $S(t)$ be the inverse local time of X at a. By defining $Y(t)$ to be the excursion of X in $(S(t-), S(t+))$ for the t value such that $S(t+) > S(t-)$, we shall obtain a point process called the *excursion point process* with values in the space of paths. Using the strong Markov property of X, we can prove that Y is a Poisson point process. Its characteristic measure, called the *excursion law*, is a σ-finite measure on the space of paths. Although it may be an infinite measure, the conditional measure, when the values of the path up to time t is assigned, is equal to the probability law of the path of the process X starting at the value of the path at t and stopped at the hitting time for a. Using this idea, we can determine the class of all possible standard Markov processes whose stopped process at the hitting time for a is a given one.

We presented this idea in our lecture at Kyoto in 1969 [3] and gave the *integral representation* of the excursion law to discuss the jumping-in case in which the excursion starts outside a. P. A. Meyer [5] discussed the general case in which continuous entering may be possible by introducing the *entrance law*. In our present paper, we will prove the *integral representation* of the excursion law in terms of the *extremal excursion laws* for the general case. It is not difficult to parametrize the extremal excursion laws by the entrance Martin boundary points for the stopped process and to determine the generator of X, though we shall not discuss it here.

E. B. Dynkin and A. A. Yushkevich [1], [2] discussed a very general extension problem which includes our problem as a special case. We shall deal with their case from our viewpoint. The excursion point process defined similarly is no longer Poisson but will be called Markov. It seems useful to study point processes of Markov type in general.

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