POTENTIAL OPERATORS FOR MARKOV PROCESSES

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1. Introduction

Yosida's definition of potential operators for semigroups [17] makes it possible to deal with transient Markov processes and a class of recurrent Markov processes in a unified operator theoretical way. In this paper, we prove some general properties of his potential operators, show which Markov processes admit the potential operators, and investigate the cases of processes with stationary independent increments as typical examples.

Let T_t be a strongly continuous semigroup of linear operators on a Banach space $\mathscr{B}$ satisfying

$$\sup_{t \geqq 0} \|T_t\| < \infty. \tag{1.1}$$

with infinitesimal generator A and resolvent

$$J_\lambda = (\lambda - A)^{-1}, \qquad \lambda > 0. \tag{1.2}$$

Following Yosida, we define potential operator V for the semigroup by

$$Vf = s \lim_{\lambda \to 0} J_\lambda f, \tag{1.3}$$

when and only when the limit exists for f in a dense subset of $\mathscr{B}$. The domain $\mathscr{D}(V)$ is the collection of f such that the limit exists. We will give conditions for the existence of the potential operator (Theorem 2.2) and prove some general properties (Theorem 2.3), summarizing Yosida's results [17], [19] with a few results added. The relation with other definitions of potential operators is shown in Theorem 2.4. In Section 3, we consider the case where $\mathscr{B}$ is the Banach space $C_0(S)$ of real valued continuous functions on S vanishing at infinity, S being a locally compact Hausdorff space with a countable base, and T_t is a semigroup induced by a Markov process transition probability. We will prove that the semigroup admits a potential operator if the Markov process is either transient or null recurrent, and that it does not admit a potential operator if the process is positive recurrent. Processes with stationary independent increments on Euclidean spaces are examined in Section 4. The fact that they admit potential operators (Theorem 4.1) is a generalization of Yosida's result [18] on Brownian motions. The domain and the representation of potential operators are