MARTIN BOUNDARIES OF RANDOM WALKS ON LOCALLY COMPACT GROUPS

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Introduction

Let G be a separable locally compact space and let (X_t), t in T, be a transient Markov process with values in G, where T is either the set of positive integers (discrete time) or the set of positive real numbers (continuous time). Let (Q^t) be the semigroup of transition kernels of (X_t). Let f and λ be, respectively, a positive Borel function on G and a positive measure on the Borel σ-field of G. Call f (respectively, λ) excessive if $Q^t f \leqq f$ and $\lim_{t \to 0} Q^t f = f$ (respectively, $\lambda Q^t \leqq \lambda$), and invariant if $Q^t f = f$ (respectively, $\lambda Q^t = \lambda$).

Around 1955, the early studies of excessive functions of a Markov process centered around two problems: the relations between Brownian motion and Newtonian potential theory, and the behavior of the trajectories of the process (X_t) as $t \to +\infty$. The latter approach can be traced back to D. Blackwell ([4], 1955) who noticed the link between bounded invariant functions and the subsets of G in which (X_t) stays, from some finite time on, with positive probability. The importance of these 'sojourn' sets became clear after W. Feller's magistral article ([24], 1956), where they are used to construct (discrete T and G) a compactification $G \cup F$ of G such that each bounded invariant function f extends continuously to $G \cup F$ and is uniquely determined by its values on the Feller boundary F.

The other approach was initiated by two papers of J. L. Doob: a study of the behavior of subharmonic functions along Brownian paths ([16], 1954), and a probabilistic approach to the potential theory of the heat equation ([17], 1955). The relation between potential theory and general (transient) Markov processes was completely clarified by G. Hunt soon after ([32], 1957–1958).

These two trends of thought each found their expression in Doob's work ([18], 1959) which revived the methods used by R. Martin ([39], 1941) in his classical study of harmonic functions. In this article, Doob constructed (for

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