ASYMPTOTIC DISTRIBUTION OF THE MOMENT OF FIRST CROSSING OF A HIGH LEVEL BY A BIRTH AND DEATH PROCESS

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1. Statement of the problem

In many applications of probability theory an essential role is played by birth and death processes, which is the name given to homogeneous Markov processes with a finite or countable number of states, which we denote by $0, 1, \cdots, n, \cdots$, in which an instantaneous transition is only possible between adjacent states. The probabilities $P_n(t) = P\{\xi(t) = n\}$ of these states satisfy the system of differential equations (see [2])

$$P_n'(t) = \lambda_{n-1}P_{n-1}(t) - (\lambda_n + \mu_n)P_n(t) + \mu_{n+1}P_{n+1}(t) \tag{1.1}$$

$n = 0, 1, \cdots$, where $\lambda_{-1} = \mu_0 = 0$.

If the number of states is finite and equals N, then $\lambda_N = \mu_{N+1} = 0$. It is also assumed that all the other parameters λ_n and μ_n are positive. Let us consider the random variable $\tau_{k,n}$, $k < n$, the passage time from state k to state n:

$$\tau_{k,n} = \inf\{t: \xi(t) = n, t > 0 | \xi(0) = k\}. \tag{1.2}$$

The random variables $\tau_{k,n}$ are of considerable interest in reliability theory, where birth and death processes describe the behavior of storage systems with replace ments. If the states $0, 1, \cdots, n - 1$, correspond to functioning states of a system, and other states correspond to nonfunctioning states of a system, then the random variable $\tau_{k,n}$ may be regarded as the length of time that the system works without a failure, if it starts in state k. Most often the state $\xi(t)$ is taken to be the number of nonfunctioning elements, at time t, in some system, and it is assumed that at the starting time the system was completely functioning, that is, $\xi(0) = 0$. Therefore, the study of the random variables $\tau_{0,n}$ is of greatest interest. Let us assume that our process has a stationary distribution. As is known [2], for this it is necessary and sufficient that the following conditions be satisfied:

$$\sum_{n=0}^{\infty} \theta_n < \infty, \qquad \sum_{n=0}^{\infty} \frac{1}{\lambda_n \theta_n} = \infty, \tag{1.3}$$