LIMIT THEOREMS FOR RANDOM WALKS WITH BOUNDARIES

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1. Introduction

In this review, we consider boundary problems for random walks generated by sums of independent items and some of their generalizations.

Let $\xi_1, \xi_2, \cdots$ be identically distributed independent random variables with distribution frunction $F(x)$. Let $S_0 = 0$, $S_n = \Sigma_{k=1}^n \xi_k$ with $n = 1, 2, \cdots$. We shall study the properties of the random trajectory formed by the sums $S_0, S_1, S_2, \cdots$. Let n be an integer parameter and let $g_n^{\pm}(t)$ be two functions on the real axis with the following properties:

$$g_n^+(0) > 0 > g_n^-(0), \qquad g_n^+(t) > g_n^-(t), \qquad t \geqq 0. \tag{1.1}$$

We shall denote by G_n the part of the halfplane $(t \geqq 0, x)$ which lies between these two curves. In the same halfplane (t, x), let us consider the trajectory formed by the points

$$\left(\frac{k}{n}, S_k\right), \qquad k = 0, 1, 2, \cdots \tag{1.2}$$

One of the main boundary functionals of trajectory (1.2) is the time η_G at which it leaves the region G_n:

$$\eta_G = \min\left\{\frac{k}{n} : \left(\frac{k}{n}, S_k\right) \notin G_n\right\}. \tag{1.3}$$

We shall define the value of the first jump χ_G across the boundary of the region G_n by the equalities

$$\chi_G = S_{\eta_G} - g_n^+(\eta_G) \quad \text{or} \quad \chi_G = S_{\eta_G} - g_n^-(\eta_G), \tag{1.4}$$

depending on whether trajectory (1.2) crosses the upper or lower boundary of the region G_n. Note that in general the random variables η_G and χ_G are not defined on the whole space of elementary events. We put $\eta_G = \infty$, where η_G remains undefined. We shall not define the random value χ_G on the set $\{\eta_G = \infty\}$.

Problems variously connected with distributions of the functionals η_G and χ_G will be called boundary problems for random walks. It is well known that these problems play an important part in mathematical statistics (in sequential analysis, nonparametric methods, and so forth) in queueing theory, and in other