POINT PROCESSES AND FIRST PASSAGE PROBLEMS

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1. Introduction and basic definitions

In many domains of application of probability theory, it becomes necessary to study various properties of random formations of points. Random sequences of arrival times in queuing systems were studied by C. Palm [17], A. Hinčin [9], F. Zitek [23], D. König, K. Matthes and K. Nawrotzki [11] and others. Statistical radiotechnica is also a source of similar problems. Here it became necessary to study the set of times corresponding to the crossing of a fixed level by a random signal (S. Rice [19], V. Tihonov [22]). It is of interest to study random point formations on the plane, on surfaces, and so forth. In this papeı, we describe a general approach which makes it possible to investigate a wide class of random point sets, generated by random processes and fields, from a common point of view.

The theory of random point sets and the random streams which correspond to them can be regarded as a special branch of the theory of random processes. However, this branch can lay claim to a certain degree of independence. Specific concepts and methods of investigation arise in it. If we are considering a random sequence of points on the line, then its definition is easily reduced to the problem of specifying a suitably defined random process. In a general setting, such an approach is frequently insufficient.

We shall consider an adequately general scheme for defining random point sets and random streams (Belyayev [2], [4]). Let $[T, \mathscr{M}_T]$ be a measurable space of values of a parameter $t \in T$, where $\mathscr{M}_T$ is the σ-algebra of measurable sets, and $[\Omega, \mathscr{F}_\Omega, P]$ is the initial probability space of elementary events $\omega \in \Omega$.

DEFINITION 1.1. *By a random stream $\eta(\Delta)$ on $[T, \mathscr{M}_T]$ is meant a random function with domain $\mathscr{M}_T$ (we denote an element of $\mathscr{M}_T$ by Δ), $\eta(\Delta) = 0, 1, \cdots, \infty$, satisfying the relation $\eta(\bigcup_i \Delta_i) = \Sigma_i \eta(\Delta_i)$ for every countable sequence $\Delta_i \in \mathscr{M}_T$ for which $\Delta_i \cap \Delta_j = \emptyset$ whenever $i \neq j$.*

DEFINITION 1.2. *A random point set defined on $[T, \mathscr{M}_T]$ is a function $S = S(\omega)$ defined on Ω, whose values are subsets of T, and such that for every $\Delta \in \mathscr{M}_T$ the number of points in $S \cap \Delta$, denoted by $\eta(\Delta)$, is a random variable. The system of all such random variables $\eta(\Delta)$ is called the random stream generated by the random point set S.*