ON A BOUND FOR THE RATE OF CONVERGENCE IN THE MULTIDIMENSIONAL CENTRAL LIMIT THEOREM

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1. Introduction

In recent years many papers concerned with estimation of the rate of convergence in the central limit theorem in R^k have appeared (see [1], [2], [6]–[8], [10], [13]–[16]). They have significantly extended our knowledge in this area. We shall mention here two recent results which are most closely related to the estimate obtained in the present paper.

V. Rotar [14], applying the method of characteristic functions, obtained a "nonuniform" estimate. It was a generalization of the corresponding one dimensional result of S. Nagaev [11] which had been extended to the case of differently distributed summands by A. Bikyalis [9]. Under the assumption that the summands are identically distributed, Rotar's result can be formulated in the following manner. If P_n is the distribution of the normalized sum $n^{-1/2}\sum_{i=1}^{n}\xi_i$ of nondegenerate, independent, identically distributed random variables with values in R^k such that $\mathscr{E}\xi_1 = 0$, $\mathscr{E}|\xi_1|^3 < \infty$, and Q is the normal distribution with the same first and second moments as ξ_1, then for any absolutely measurable convex set $E \subset R^k$

$$|P_n(E) - Q(E)| \leqq c(k)\frac{\mathscr{E}(\Delta^{-1}\xi_1, \xi_1)^{3/2}}{1 + \imath_\Delta^3(E)}\, n^{-1/2}, \tag{1.1}$$

where $c(k)$ depends only on k, Δ is the covariance matrix of ξ_1 and $\imath_\Delta(E)$ is defined in formula (3.2) below.

On the other hand, V. Paulauskas [13], applying the method of composition of H. Bergström [3]–[6] and using the results of the author [16], derived a bound in terms of "pseudo moments" which, in the notation introduced above, takes the form

$$|P_n(E) - Q(E)| \leqq c(k)\nu_3' n^{-1/2}, \tag{1.2}$$

where

$$\nu_3' = \max(\nu_3, \nu_3^{1/4}), \qquad \nu_3 = \int_{R^k} (\Delta^{-1}x, x)^{3/2}|P - Q|(dx). \tag{1.3}$$

(Here $|P - Q|$ denotes the variation of the measure $P - Q$.)