CENTRAL LIMIT THEOREM FOR STATIONARY PROCESSES

M. ROSENBLATT
UNIVERSITY OF CALIFORNIA, SAN DIEGO

1. Introduction

A discussion of strong mixing and uniform ergodicity is presented, partly in terms of their relation to the central limit problem. Some of the gaps in one's understanding of the proper domain of validity of the central limit theorem for stationary sequences are pointed out. A definition of strong mixing appropriate for stationary random fields is given. A version of a limit theorem for stationary random fields with asymptotic normality is then derived. The argument for this limit theorem uses martingalelike ideas.

2. Stationary sequences

By this time there is an extensive literature on the central limit theorem for stationary processes, especially with respect to asymptotic normality. However, much of this is still rather unsatisfactory since it leads to effective computational results only under limited circumstances. We shall give a brief sketch of some of the ideas that have been used. For convenience, discrete time stationary processes will be discussed for the most part since the case of continuous time parameter processes can usually be easily reduced to the discrete time case.

Let $\{X_n, n = \cdots, -1, 0, 1, \cdots\}$ be a discrete time parameter stationary process. The Borel fields $\mathscr{B}_n = \mathscr{B}(X_k, k \leqq n)$, $\mathscr{F}_m = \mathscr{B}(X_k, k \geqq m)$ are generated by the random variables up to time n and from time m, respectively. They represent the past relative to n and future relative to m, respectively. A condition called strong mixing was proposed in [12] and amounted to

$$\sup_{B \in \mathscr{B}_0, F \in \mathscr{F}_n} |P(BF) - P(B)P(F)| \to 0 \tag{2.1}$$

as $n \to \infty$ where P is the probability measure of the stationary process. The condition has interest on its own but it was originally proposed together with some additional moment conditions to get asymptotic normality for partial sums of the random variables of a process properly normalized. A later version of such a central limit theorem using strong mixing can be found in Ibragimov's paper [6]. However, the condition (2.1) also has an amusing alternative interpretation

This research was supported by the Office of Naval Research, Grant No. N00014-69-A-0200-6016.