

# LIMIT THEOREMS FOR SUMS OF A RANDOM NUMBER OF POSITIVE INDEPENDENT RANDOM VARIABLES

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## 1. Statement of the problem

The work of A. Wald [11] and H. Robbins [10] played an important role in stimulating interest in the investigation of sums of a random number of independent random variables. Of late a large number of papers related to this topic have been published and their importance for numerous applied questions as well as theoretical mathematics has been shown.

Results of research, conducted over the last three years by myself and my students in connection with problems in the theory of reliability, theory of queuing, physics, and production organization, are presented in this paper. We shall begin the presentation with the consideration of one of these problems. Some of the formulations of problems and theorems which will be included here have been presented before when I delivered lectures at the Universities of London, Sheffield, Rome, Budapest and Warsaw where I was a guest in recent years. Several statements of problems and their solutions originated there.

Geiger-Müller counters are used in nuclear physics and also in the study of cosmic radiation. A particle having struck the counter and having been counted by it causes a breakdown in it lasting some time  $\tau$ . Any particle striking the counter in the period of the breakdown is not registered by it. Usually one assumes that  $\tau$  is a constant; however, a more realistic assumption is that  $\tau$  is a random variable with some distribution  $G(x)$ . Our problem is to determine the distribution of the length of the time interval from the first registration of a particle to the first loss. Here we shall assume that the time intervals between successive entries of particles are independent and identically distributed with distribution function  $F(x)$ .

It is obvious that the counter does not lose a particle until the duration of the breakdown is as large as the interval between successive entries of particles. (See Figure 1.) This permits us to write the following equality:

$$(1.1) \quad \eta = \xi_1 + \xi_2 + \cdots + \xi_\nu,$$

where  $\xi_k$  is the time between the arrival of the  $k$ th particle and the  $(k + 1)$ st particle and  $\nu = \min \{k, \xi_k < \tau_k\}$ .