ASYMPTOTIC NORMALITY FOR SUMS OF DEPENDENT RANDOM VARIABLES

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1. Introduction

1.1. Limiting distributions of sums of independent random variables have been exhaustively studied and there is a satisfactory general theory of the subject (see the monograph of B. Gnedenko and A. Kolmogorov [6], or advanced text books on probability theory such as that of M. Loève [10]). Our knowledge of the corresponding theory for dependent random variables is much more meagre. Although a great number of papers have been published on the subject, not many general results are known. In recent years the author has shown ([4], [5]) that the necessary and sufficient conditions for convergence in distribution to any specified, infinitely divisible law remain sufficient also in the most general dependent case, provided that quantities such as means and the like, are replaced by conditional means, and the like, the conditioning being relative to the preceding sum. (The necessity of the conditions requires, in general, further assumptions.)

In the present paper we are concerned almost exclusively with asymptotic normality. Though our general results about asymptotic normality can be obtained by direct specialization of the results mentioned above, we preferred to develop them here independently. We hope that the greater accessibility of the present proofs will compensate for this sacrifice of brevity.

After establishing the general results we give a few applications. It would be quite easy to extend the list of applications indefinitely by going through various results in the literature and seeing how they can be improved by using our general theorems.

1.2. We consider random variables arranged in a double array

$$\begin{array}{l} X_{1,1}, X_{1,2}, \cdots, X_{1,k_1} \\ X_{2,1}, X_{2,2}, \cdots, X_{2,k_2} \\ \vdots \\ X_{n,1}, X_{n,2}, \cdots, X_{n,k_n} \\ \vdots \end{array} \tag{1.1}$$