THE CENTRAL LIMIT THEOREM FOR MARKOV PROCESSES

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1. Introduction

The central limit theorem has been presented at various levels of generality and application, both with respect to types of processes considered and to types of limit laws considered. The main results of this paper, contained in Section 4, refer to Markov processes, with a time homogeneous law of evolution, whose transition probabilities, averaged in the Cesaro (C-1) sense, converge to a common limit. As indicated in Theorem 2.1, this is equivalent to the assumption that the process has a finite invariant measure and the state space is a final set (in the sense of Doeblin [5]). The results can be extended to processes on indecomposable sets and this generalization is indicated. Conditions for convergence to any infinitely divisible law are given, and special consideration is given to normal and clustering (compound Poisson) distributions.

The method is related to the familiar approach when the state space is countable: look at the interblocks between successive returns to a given state, these being independent with a common distribution when the process is started at this state. For a noncountable state space the single state of the classical approach must be replaced by a uniform state set, a notion first used in [4]. Section 2 provides an introduction to the terminology used regarding Markov processes, in particular to uniform state sets. The interblocks between successive returns to a uniform state set are identically distributed for the process with a suitable starting distribution, but are no longer independent. They do, however, satisfy a pointwise strong mixing condition.

Thus, we need the central limit theorem for pointwise strong mixing stationary sequences. This was first presented in its general form in [3]. In Section 3 a revised and expanded version is developed, which is suitable to the present application.

This study is based upon the comparison of laws of functionals of the process to laws of sums of related independent random variables. This requires the notion of asymptotic equivalence of laws developed by M. Loève; a brief introduction to the subject is provided at the end of Section 2.

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