# GROWTH RATE OF CERTAIN GAUSSIAN PROCESSES

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## 1. Introduction

We will be concerned with real, continuous Gaussian processes. In (A) of Theorem 1.1, a result on the growth rate of the supremum as $t \to \infty$ is given. The processes covered by Theorem 1.1 all have stationary increments. The law of the iterated logarithm, given as (C) below, is a consequence of (A).

Theorem 1.1 will be stated and discussed in this section. The proof of this theorem and supporting propositions are given in Section 2. An analogous result for small times is given in Section 3. That the method of proof can also be successfully employed in dealing with certain Gaussian processes not possessing stationary increments is illustrated by Theorem 4.1. The results of Section 1 were announced in [5].

Let $(Y_t, t \geqq 0)$ be a real, separable Gaussian process with $Y_0 = 0, E[Y_t] \equiv 0$, and set

$$w(s, t) = E[Y_s Y_t],\ Q(t) = \tfrac{1}{2} w(t, t). \tag{1.1}$$

Then let

$$X_t = \frac{Y_t}{(2Q(t))^{1/2}}. \tag{1.2}$$

In this section and the succeeding two sections, $(Y_t)$ will be taken to have stationary increments, so that

$$w(s, t) = Q(s) + Q(t) - Q(t - s), \qquad 0 \leqq s \leqq t. \tag{1.3}$$

THEOREM 1.1. *Suppose there exists a monotone nondecreasing function* $v(t)$, *defined on the nonnegative reals and vanishing at* $t = 0$, *and there exist positive constants* $s_0, \beta_1, \beta_2, \beta_3$, *with* $\beta_3 < \frac{1}{2}\beta_1 + 1$, *such that*

$$\lim_{t \to \infty} \frac{Q(s + t) - Q(s)}{v(s + t) - v(s)} = 1 \text{ uniformly in } s, \tag{1.4}$$

*and*

$$v(t) \geqq \left(\frac{t}{s}\right)^{\beta_1} v(s) > 0, \qquad t \geqq s > s_0, \tag{1.5}$$

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