SUPPORTS OF GAUSSIAN MEASURES

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1. Introduction

The present paper is a continuation of the work, carried out in [4] and [5] of investigating the relationship between a Gaussian process and its reproducing kernel Hilbert subspace. Our main result gives a characterization of the topological support of a Gaussian measure defined on a linear topological space of functions on an arbitrary set. As special cases we consider Gaussian processes on Banach spaces and on duals of Fréchet spaces.

2. Preliminary results

In this section we state, and in some cases prove, several results concerning the equivalence of Gaussian measures under translation. Many of these are modifications of well known results. They are included here since the modified versions (for the most part having to do with the removal of separability requirements) do not seem to be available in the literature. The most important result is Theorem 2.2, which provides us with the basic technique for the proofs of our support theorems. It was obtained as Lemma 6 of [4] and used there to derive certain zero-one laws for Gaussian processes.

Let T be a nonempty set, X a linear space of real valued functions on T and $\mathscr{A} = \mathscr{A}(X: T)$ the smallest σ-field of subsets of X under which all the evaluation maps $x \to x(t)$, $t \in T$, are measurable. Let a Gaussian probability measure P_0 be given on $\mathscr{A}$ such that its mean function $Ex(t) = 0$ for all t in T and

$$R(t, s) = \int_X x(t)x(s)P_0(dx) \tag{2.1}$$

is its covariance kernel. The symbol $\mathscr{A}_0 = \mathscr{A}_0(X; T)$ will denote the completion of $\mathscr{A}$ with respect to P_0. Let $H(R)$ be the reproducing kernel Hilbert space (RKHS) determined by R. For the definition of a RKHS see [4] where further references are given. We shall assume that $H(R)$ is a space of functions on T and that the basic space X is rich enough to contain $H(R)$.

If S is any countable subset of T, write $\mathscr{A}_S = \mathscr{A}(X; S)$ for the smallest σ-field of subsets of X with respect to which the maps $x \to x(t)$, $t \in S$, are measurable,

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