CONTINUITY PROPERTIES OF GAUSSIAN PROCESSES WITH MULTIDIMENSIONAL TIME PARAMETER

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In this paper we shall present a strengthening and generalization to higher dimensions of the real variable lemma presented in [4].

As a consequence we shall obtain a criterion for the continuity of sample functions of Gaussian processes with a multidimensional time parameter.

Remarkably enough, the difficulty of the arguments here is almost independent of dimensions, indeed the proofs in this paper are considerably simpler and yield stronger results than those in [4].

As in [4] our point of departure is a real variable lemma giving an *a priori* modulus of continuity for functions satisfying certain integral inequalities.

As in [4], the basic ingredients are two functions $p(u)$, defined in $[-1, 1]$ and $\Psi(u)$, defined in $(-\infty, +\infty)$. However here, in addition to the conditions

$$p(u) = p(-u) \downarrow 0 \quad \text{as} \quad |u| \downarrow 0, \tag{1}$$

$$\Psi(u) = \Psi(-u) \uparrow \infty \quad \text{as} \quad |u| \uparrow \infty, \tag{2}$$

we shall assume that $\Psi(u)$ is *convex*.

Let then I_0 denote the unit hypercube in d dimensional cartesian space. In this paper, by "hypercube" we mean a hypercube with edges parallel to the coordinate axes.

For every hypercube I we denote by $|I|$ its volume and by $e(I)$ the common length of its edges.

This given, we can state our basic lemma in the following form.

LEMMA 1. *Let $f(x)$ be measurable in I_0 and such that*

$$\int_I \int_I \Psi\left(\frac{f(x) - f(y)}{p(e(I))}\right) dxdy \leqq B, \qquad \textit{for all } I \subset I_0. \tag{3}$$

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