SETS OF BOUNDEDNESS AND CONTINUITY FOR THE CANONICAL NORMAL PROCESS

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1. Introduction

Let X be a linear space; by a *linear stochastic process* will be meant a map L from X to the linear space of measurable functions on some probability space $(\Omega, S, \Pr)$ such that, for each real a, b and each x, y in X, $aL_x + bL_y = L_{ax+by}$ with probability one. Two such maps are equivalent if they have the same finite dimensional joint distributions. Such processes have been objects of active investigation in recent years, both for their intrinsic interest and in connection with the study of other stochastic processes. For background, we refer the reader to [2].

A central role is played by the *canonical normal processes* on real Hilbert spaces, which are characterized by the conditions that each L_x is Gaussian with zero mean and $E\{L_x L_y\} = (x, y)$. The restrictions of these processes provide models for all Gaussian processes with zero mean, in the following way. Let $\{\xi_t, t \in T\}$ be a Gaussian process with zero mean. Let H be the closed subspace of $L^2(\Pr)$ spanned by the random variables of the process. Thus, a map $\phi : T \to H$ is defined. Let L be the map $: H \to L^0(\Pr)$ which assigns to each $x \in H$ the selfsame random variable regarded as a member of $L^0(\Pr)$. Then L is a linear map $: H \to L^0(\Pr)$, that is, a linear stochastic process on H. Each L_x is Gaussian with zero mean, and $E(L_x L_y) = (x, y)$, so that L is a version of the canonical normal process on H. The original process ξ is then given by $\xi_t = L_{\phi(t)}$.

Recently this viewpoint has been developed, notably in [2] and [8], in connection with the study of pathwise boundedness and pathwise continuity of Gaussian processes. It is easy to see that ξ will have a pathwise bounded version if and only if L has a version whose restriction to $\phi(T)$ is pathwise bounded. If T has a topology, and $t \rightsquigarrow \xi_t$ is stochastically continuous, then ϕ is continuous: $T \to H$, so the existence of a version of L whose restriction to $\phi(T)$ is pathwise continuous will imply the existence of a pathwise continuous version for ξ; while if ϕ is also an open map, as is the case when T is compact, then the converse implication also holds.

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