STRICTLY ERGODIC SYMBOLIC DYNAMICAL SYSTEMS

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1. Introduction

We continue the study of strictly ergodic symbolic dynamical systems which was started in our earlier report [6]. The main tools used in this investigation are "homomorphisms" and "substitutions". Among other things, we construct two strictly ergodic symbolic dynamical systems which are weakly mixing but not strongly mixing.

2. Strictly ergodic symbolic dynamical systems

Let A be a finite set consisting of more than one element. Let

$$X = A^Z = \prod_{n \in Z} A_n, \qquad A_n = A \quad \text{for all } n \in Z, \tag{2.1}$$

be the set of all two sided infinite sequences

$$x = \{a_n \mid n \in Z\}, \qquad a_n = A \quad \text{for all } n \in Z, \tag{2.2}$$

where

$$Z = \{n \mid n = 0, \pm 1, \pm 2, \cdots\} \tag{2.3}$$

is the set of all integers. For each $n \in Z$, a_n is called the nth *coordinate* of x, and the mapping

$$\pi_n : x \to a_n = \pi_n(x) \tag{2.4}$$

is called the nth *projection* of the *power space* $X = A^Z$ onto the *base space* $A_n = A$. The space X is a totally disconnected, compact, metrizable space with respect to the usual direct product topology.

Let φ be a one to one mapping of $X = A^Z$ onto itself defined by

$$\pi_n(\varphi(x)) = \pi_{n+1}(x) \qquad \text{for all } n \in Z. \tag{2.5}$$

The mapping φ is a homeomorphism of X onto itself and is called the *shift transformation*. The dynamical system (X, φ) thus obtained is called the *shift dynamical system*.

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