INEQUALITIES ON THE PROBABILITY CONTENT OF CONVEX REGIONS FOR ELLIPTICALLY CONTOURED DISTRIBUTIONS

S. DAS GUPTA
UNIVERSITY OF MINNESOTA

M. L. EATON
UNIVERSITY OF CHICAGO

I. OLKIN
STANFORD UNIVERSITY

M. PERLMAN
UNIVERSITY OF MINNESOTA

L. J. SAVAGE
YALE UNIVERSITY

M. SOBEL
UNIVERSITY OF MINNESOTA

We dedicate this paper to the memory of L. J. Savage.

1. Introduction

Let $x = (x_1, \cdots, x_p)$ be a p dimensional random vector,

$$(1.1) \quad P(\Sigma) \equiv P_\Sigma\{|x_1| \leqq h_1, \cdots, |x_p| \leqq h_p\} = \int_{-h}^{h} |\Sigma|^{-1/2} f(x\Sigma^{-1}x')\, dx,$$

and

$$(1.2) \quad P^+(\Sigma) \equiv P_\Sigma\{x_1 \leqq \ell_1, \cdots, x_p \leqq \ell_p\} = \int_{-\infty}^{\ell} |\Sigma|^{-1/2} f(x\Sigma^{-1}x')\, dx,$$

where $h = (h_1, \cdots, h_p)$ and $\ell = (\ell_1, \cdots, \ell_p)$ are constant vectors, $h_i \geqq 0$, $i = 1, \cdots, p$, and $\Sigma = (\sigma_{ij})$ is a positive definite matrix. We call a density (with respect to Lebesgue measure) of the form

$$(1.3) \quad |\Sigma|^{-1/2} f(x\Sigma^{-1}x'),$$

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