ON THE SPAN IN L^p OF SEQUENCES OF INDEPENDENT RANDOM VARIABLES (II)

HASKELL P. ROSENTHAL
UNIVERSITY OF CALIFORNIA, BERKELEY

1. Introduction

This work is motivated by the following problem: which Banach spaces are isomorphic (linearly homeomorphic) to a complemented subspace of $L^p(= L^p[0, 1])$ for $1 < p < \infty$, $p \neq 2$, and what are their linear topological properties? (A linear subspace A of a Banach space B is said to be complemented if there exists a bounded linear operator P on B with range A such that $P^2 = P$; such a P is called a projection onto A.) It is well known that ℓ^2, ℓ^p, $\ell^2 \oplus \ell^p$, $(\ell^2 \oplus \ell^2 \oplus \cdots)_p$, and of course L^p itself, are examples of such Banach spaces. As usual, ℓ^p denotes the Banach space of sequences of scalars (x_n) such that $(\Sigma|x_n|^p)^{1/p} = \|(x_n)\| < \infty$; given Banach spaces $B_1, B_2, \cdots$; we denote by $(B_1 \oplus B_2 \oplus \cdots)_p$ the Banach space of all sequences (b_n) such that $b_n \in B_n$ for all n and $\|(b_n)\| = (\Sigma\|b_n\|^p)^{1/p} < \infty$. Given a sequence (b_n) of elements of a Banach space B, we denote by $[b_n]$ the closed linear span of its terms; if $B = L^p$ of some probability space, then $[b_n]$ is denoted also by $[b_n]_p$.

To see that Hilbert space, that is, ℓ^2 is an example, one may consider a sequence (f_n) of two valued, symmetric, independent random variables; it follows from Khintchine's inequalities that $[f_n]_p$ is isomorphic to Hilbert space. Moreover, the 2 and p norms are equivalent on $[f_n]_p$, and orthogonal projection onto $[f_n]_2 = [f_n]_p$ shows that $[f_n]_p$ is complemented.

Now let $2 < p < \infty$, and let (f_n) be a sequence of independent, nonzero random variables belonging to L^p, each of mean zero. We proved in [11] that $[f_n]_p$ is isomorphic to a complemented subspace of L^p, and that $[f_n]_p$ is isomorphic to exactly one of four Banach spaces: ℓ^2, ℓ^p, $\ell^2 \oplus \ell^p$, or a new space which we denote as $\mathbf{X}_p$. We showed moreover that if the f_n are three valued, symmetric, then $[f_n]_p$ is complemented in L^p by means of orthogonal projection, with $[f_n]_q$ thus complemented and isomorphic to $([f_n]_p)^*$, the dual of $[f_n]_p$ (throughout, p and q used together, denote reals satisfying $1/p + 1/q = 1$). We thus obtained that $\mathbf{X}_q$ (defined as the dual of $\mathbf{X}_p$) is isomorphic to the span of a sequence of independent random variables in L^q, providing the starting point for the present

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