ON A CLASS OF MOMENT PROBLEMS

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Introduction

Consider a class $\mathcal{M}$ of probability measures on a measurable space X and measurable functions g_j and h on X. In a typical moment problem we want further information on the possible sets of values taken by the integrals $\mu(g_j)$ and $\mu(h)$ as μ runs through $\mathcal{M}$. And the main purpose of the present paper is to develop into more systematic methods certain principles which in special cases have been found effective for handling such moment problems.

In Sections 2 through 4 we take up certain frequently occurring moment problems where the class $\mathcal{M}$ happens to be convex. In Section 2 the space X can be any locally compact Hausdorff space. For $\{h_j, j \in J\}$ as an arbitrary collection (finite or infinite) of lower semicontinuous functions on X, we establish a condition which is both necessary and sufficient for the existence of a regular probability measure μ on X satisfying $\mu(h_j) \leqq \eta_j$ for all $j \in J$. However, we do assume as a side condition that the h_j dominate each other at infinity in a certain weak sense. This domination condition is void when X is compact and nearly so when $h_j \geqq 0$.

In Sections 3 and 4 we are interested in the smallest value $L(y)$ of $\mu(h)$ when it is known that $\mu \in \mathcal{M}$ and that $\mu(g_j) = y_j$ for $j = 1, \cdots, n$; the space X can be any measurable space. Provided this smallest value $L(y)$ is in fact assumed, it turns out that in the determination of $L(y)$ we only need to consider so called *admissible* measures.

These are defined as the measures $\mu \in \mathcal{M}$ which attain the smallest possible value $\mu(\psi)$ for some linear combination ψ of the form $\psi = h - d_1 g_1 - \cdots - d_n g_n$. In the special case that $\mathcal{M}$ consists of all probability measures on X, we have admissibility if and only if the measure is carried by the set of minima of some such linear combination ψ.

In Sections 5 and 6 we are interested in bounds for and inequalities between the different moments of a sum $S_n = Z_1 + \cdots + Z_n$ of independent random variables Z_i. Here, the Z_i may have different distributions subject to certain restrictions on these distributions. The resulting collection $\mathcal{M}$ of possible distributions of S_n is usually not convex.

An essential use is made of the fact that each cumulant $\kappa_j(S_n)$ of S_n is equal to the sum of the $\kappa_j(Z_i)$. The set $K[q]$ of possible q-tuples $(\kappa_1(Z), \cdots, \kappa_q(Z))$ is usually not a convex subset of R^q. It turns out that for large n the existing

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