LIFTINGS COMMUTING WITH TRANSLATIONS

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1. Introduction

Let Z be a locally compact group and β a (left invariant) Haar measure on Z. We denote by $M_R^\infty(Z, \beta)$ the Banach algebra of all bounded real valued measurable functions defined on Z, endowed with the norm

$$f \rightsquigarrow \|f\|_\infty = \sup_{z \in Z}|f(z)|, \tag{1.1}$$

and by $C_R^b(Z)$ the subalgebra of all continuous bounded real valued functions defined on Z.

For two functions f and g, with domain Z, we write $f \equiv g$, whenever f and g coincide $\beta^\bullet$ almost everywhere.

We denote by $\mathscr{B}$ the *tribe* of all measurable parts of Z and by $\mathscr{B}_0$ the *clan* of all $A \in \mathscr{B}$ satisfying $\beta^\bullet(A) > +\infty$.

A mapping $\rho: M_R^\infty(Z, \beta) \to M_R^\infty(Z, \beta)$ is a lifting of $M_R^\infty(Z, \beta)$ if:

(I) $\rho(f) \equiv f$;

(II) $f \equiv g$ implies $\rho(f) = \rho(g)$;

(III) $\rho(1) = 1$;

(IV) $f \geqq 0$ implies $\rho(f) \geqq 0$;

(V) $\rho(\alpha f + \beta g) = \alpha\rho(f) + \beta\rho(g)$;

(VI) $\rho(fg) = \rho(f)\rho(g)$.

For $s \in Z$ and $f: Z \to R$ we denote $\gamma(s)f$ the mapping $z \rightsquigarrow f(s^{-1}z)$ of Z into R. A lifting ρ of $M_R^\infty(Z, \beta)$ commutes with (the left translations of) Z if

(VII) $\rho(\gamma(s)f) = \gamma(s)\rho(f)$

for all $s \in Z$ and $f \in M_R^\infty(Z, \beta)$.

In the paper [1], published in the Proceedings of the Fifth Berkeley Symposium, it has been shown that *for every locally compact group Z there exists a lifting of $M_R^\infty(Z, \beta)$ commuting with Z* and that such a lifting is strong. In the next two sections we shall give two applications of this result.

This paper was written under contract DAHCOH 68 C005 with the U.S. Army Research Office, Durham, N.C.