METRIC MEASURE SPACES OF ECONOMIC AGENTS

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1. Introduction

An economic agent, who participates in a pure exchange economy is described by his *needs*, *tastes*, and *endowments*. If there are ℓ commodities, these economic concepts are formalized as follows:

the needs are described by a subset $X \subset R^{\ell}$; in choosing a commodity vector, the agent is restricted to the set X;

the tastes are described by a binary relation $\precsim$ on X; $x \precsim y$ means that the commodity vector y is at least as desired as the commodity vector x;

the endowments are described by a vector e in the commodity space R^{ℓ} (for more details see Chapters 2 and 4 of G. Debreu [6]).

A *pure exchange economy* is a finite family $\{(X_a, \precsim_a, e_a)\}_{a \in A}$ of economic agents. Since the endowments e_a typically are not a maximal element for $\precsim_a$ in X_a, there is an incentive to exchange commodities in order to improve the initial position.

The result of an exchange is a redistribution of the total endowments; it can be described by a function f of A into R^{ℓ} such that for every $a \in A$, $f_a \in X_a$ and $\Sigma_A f_a = \Sigma_A e_a$ or, if we assume free disposal of all commodities, $\Sigma_A f_a \leqq \Sigma_A e_a$.

The economic analysis of pure exchange economies consists of specifying a certain class of redistributions as possible outcomes of the exchange process.

Imagine a planner who cannot enforce his plan and who proposes a certain redistribution f. If there exists a subset B of agents and for every agent a in B a commodity vector $g_a \in X_a$ such that g_a is preferred to f_a by every member a in B and $\Sigma_B g_a \leqq \Sigma_B e_a$, then the coalition B has the desire and the power to block the proposed plan f. It seems reasonable to exclude as possible outcomes all redistributions which can be blocked by any coalition. The remaining redistributions are called the *core* of the exchange economy. The core, if not empty, contains in general many redistributions.

Suppose now that all agents agree to exchange commodities in fixed ratios. This agreement simplifies the exchange extremely. If $p \in R^{\ell}$ is the price system (that is, p_h/p_k is the amount of commodity k one has to give for one unit of commodity h), the agent $a = (X, \precsim, e)$ will only consider vectors in his budget set $\{x \in X | p \cdot x \leqq p \cdot e\}$ and will choose a most desired vector in this set. A redistribution f and a price vector p is called a *price equilibrium*, if the commodity vector f_a is for every agent a most desired vector in his budget set. The existence of price equilibria can be shown under quite general conditions (Debreu [6]).