PROJECTIVE LIMITS OF MEASURE SPACES

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1. Introduction

This paper introduces analytic measures and proves several theorems on projective limits of analytic measures. The usual existence theorems assume either strong conditions on the conditional probabilities, or compactness of σ-measures and "continuity" of mappings (see Theorem 4.1 below). This is an attempt to assume something about measurable spaces only, and nothing about mappings. More explicitly, we want to find a theorem for general systems of measure spaces similar to the classical theorem that says the indirect product of perfect σ-measures always exists.

Throughout this paper, we consider a measure to be a finitely additive, finite, nonnegative measure and σ-additive measures are called σ-measures. It should be remarked that all results hold for σ-finite measures; the proof of that generalization is trivial.

In Section 2, the problem of the existence of the projective limit of σ-measure spaces is described and notation is introduced.

In Section 3, the "morphisms" between systems of measure spaces are introduced and applications to existence problems are mentioned. In Sections 2 and 3, the use of category theory may be helpful, however, no results of category theory are assumed—what is needed is explained without any sophistication.

In Section 4, the existence results of S. Bochner [3], J. Choksi [4] and M. Métivier [24] are recalled in Theorem 4.1.

In Section 5, the relevant properties of analytic spaces are recalled and the main results are proved, that is Theorems 2.2, 5.4, 5.5, 5.6, and 5.7. In conclusion, a short survey of analytic spaces is given.

2. Presheaves of measure spaces

2.1. Let $\langle I, \leqq \rangle$ be a directed set (that is, $\leqq$ is a reflexive order on I such that each finite subset of I has an upper bound), and let

$$(2.1) \qquad \langle \{\langle X_n, \mathscr{B}_n, \mu_n \rangle\}, \{f_{n,m} \mid n \geqq m\} \rangle$$

be a presheaf of measure spaces (over $\langle I, \leqq \rangle$). That means each