THE RADON-NIKODÝM DERIVATIVE OF A CORRESPONDENCE

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1. Introduction

Let $(A, \mathscr{A}, \nu)$ be a complete, totally σ-finite, positive measure space and S be an ordered finite dimensional real vector space with its usual topology and the Borel σ-field $\mathscr{S}$ generated by this topology. Given a function γ from A to $\mathscr{P}(S)$, the set of subsets of S, we define its integral over $E \in \mathscr{A}$ by

$$(1.1)\qquad \int_E \gamma d\nu = \{x \in S \mid \text{there is an integrable function } f \text{ from } E \text{ to } S \text{ such that } x = \int_E f d\nu \text{ and a.e. in } E, f(a) \in \gamma(a)\}.$$

And given a function Γ from $\mathscr{A}$ to $\mathscr{P}(S)$, we say that a function γ from A to $\mathscr{P}(S)$ is a Radon-Nikodým derivative of Γ if

$$(1.2)\qquad \text{for every } E \in \mathscr{A},\ \Gamma(E) = \int_E \gamma d\nu.$$

When $\Gamma(E)$ is nonempty for every $E \in \mathscr{A}$, we call Γ a correspondence from $\mathscr{A}$ to $\mathscr{S}$. In this article we characterize the correspondences from $\mathscr{A}$ to S, having a measurable, positive, closed, convex valued Radon-Nikodým derivative, where a function γ from A to $\mathscr{P}(S)$ is defined as measurable if its graph

$$(1.3)\qquad G(\gamma) = \{(a, x) \in A \times S \mid x \in \gamma(a)\}$$

belongs to the product σ-field $\mathscr{A} \otimes \mathscr{S}$.

The need for such a characterization arose in the theory of economic systems in which certain sets of negligible agents are not negligible. To describe this situation mathematically one introduces a set A of agents, a σ-field $\mathscr{A}$ of subsets of A (the σ-field of coalitions), and a positive measure ν defined on $\mathscr{A}$. Now the

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