# WILLIAM FELLER AND TWENTIETH CENTURY PROBABILITY

## 1. Twentieth Century Probability

When William Feller was born in 1906, Lebesgue measure had just been invented, and Fréchet was to introduce measure on an abstract space about ten years later. Thus, the technical basis of modern mathematical probability was developed about the time of Feller's early childhood. Since that time the subject has been transformed, by no one more than by Feller himself, into an essential part of mathematics, contributing to other parts as well as drawing from them.

In the first part of our century, few probabilists felt comfortable about the basis of their subject, either as an applied or as a purely mathematical subject. In fact, it was commonly judged that there was no specific mathematical subject "probability," but only a physical phenomenon and a collection of mathematical problems suggested by this phenomenon. A probabilist joked that probability was "a number between 0 and 1 about which nothing else is known." In the discussions of the foundations of probability, there was no clear distinction made between the mathematical and the real. For example, one influential theory was that of von Mises, based on the concept of a "collective," which was defined as a sequence of observations with certain properties. Since "observation" is not a mathematical concept and since the properties were properties which no mathematical sequence could have, the theory could survive in its original form only by an affirmation that it was not a formal mathematical theory but an attempt at a direct description of reality. Instead, the theory was restricted to remove the mathematical objection, unfortunately losing in intuitive content what it gained in mathematical significance. The fate of the theory was an inevitable result of the increasing demand of mathematicians for exact definitions and formal rigor. The present formal correctness of mathematical probability only helps indirectly in analyzing real probabilistic phenomena. It is unnecessary to stress to statisticians that the relation between mathematics and these phenomena is still obscure. Or if not obscure it is clear to many but in mutually contradictory ways.

Formalizations of mathematical probability by Steinhaus in 1923 and Fréchet in 1930 were too incomplete to have much influence. The first acceptable formalization was by Kolmogorov in his 1933 monograph. Of course before that, and in fact for at least two centuries before that, there had been mathematicians who made correct and valuable contributions to mathematical probability. Mathematicians could manipulate equations inspired by events and expectations before these concepts were formalized mathematically as measurable sets and integrals. But deeper and subtler investigations had to wait until the blessing and curse of direct physical significance had been replaced by the bleak reliability of abstract mathematics.