EXACT SIGNIFICANCE TESTS FOR CONTINGENCY TABLES EMBEDDED IN A 2^n CLASSIFICATION

MARVIN ZELEN
STATE UNIVERSITY OF NEW YORK AT BUFFALO

1. Introduction

This paper considers the analysis of multidimensional contingency tables when the contingency table can be regarded as being embedded in a 2^n factorial classification. The model assumes that the response variable is binary and is observed over n factors each at two levels. This gives rise to 2^{n-1} 2×2 contingency tables. The theoretical development is in the spirit of the Fisher-Irwin treatment of the 2×2 table. The work reported here can be regarded as a generalization and extension of their work.

The new techniques for analyzing contingency tables derived here are based on conditional reference sets. This allows derivation of exact tests of significance for testing interactions arising in a contingency table context. These tests are conditional tests and have the property that they are uniformly most powerful unbiased tests.

Although this paper only discusses binary response random variables embedded in a 2^n classification, the methods are readily extended to multinomial response embedded in an arbitrary cross classification structure. In a later paper, analyses for more general contingency tables will be developed.

The classical method for analyzing the interactions associated with a complex classification is based on chi square goodness of fit tests. More recently Kullback and his associates [5], [6] have used the ideas of information theory to analyze multidimensional contingency tables. These techniques are equivalent to likelihood ratio tests. However, both the chi square and likelihood ratio techniques are based on asymptotic distributions. The methods of analysis which use a logit model or a multiplicative model for the probability of a response also are based on asymptotic theory. It is interesting that recent reviews of the analysis of contingency tables do not refer to any exact tests for testing interactions (see Lewis [10], Goodman [4], and Plackett [11]).

This work was supported by Public Health Service Research Grant No. CA-10810 from the National Cancer Institute.