SOME MULTIPLICATIVE MODELS FOR THE ANALYSIS OF CROSS CLASSIFIED DATA

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1. Introduction and summary

In the present article, we shall present some multiplicative models for the analysis of $R \times C$ contingency tables (that is, contingency tables with R rows and C columns), and shall apply these models to cross classified data in ways that will lead to a more complete analysis of these data than has heretofore been possible.

For the $R \times C$ contingency table, the usual model of "independence" in the table (that is, independence between the row classification and column classification of the table) is a simple example of a multiplicative model. For short, I shall call this model (that is, the model of independence between the row classification and column classification of the table) the I model. The model of "quasi-independence" in the $R \times C$ table, which was introduced and developed in my earlier work, and which I shall comment upon again later, is another example of a multiplicative model (see, for example, Goodman [9], [10], [12], [13], Caussinus [4], Bishop and Fienberg [2]). For short, I shall call this model the Q model. The various multiplicative models which I shall present here can be viewed as modifications or generalizations of the I model and/or the Q model.

To illustrate the application of these models, we shall analyze a 5×5 contingency table (Table I) in which there is a one to one correspondence between the five classes of the row classification and the five classes of the column classification, and in which the classes of the row (and column) classification can be ordered (from high to low). Although some of the particular models, which we shall consider herein (see Section 2), are particularly well suited to square contingency tables of this kind (in which there is this one to one correspondence and in which the classes of the row (and column) classification can be ordered), we wish to draw the reader's attention to the fact that the general class of multiplicative models presented in this article (see Sections 3 and 4) also includes a variety of models that can be applied more generally to rectangular contingency tables (as well as to square tables), where there may or may not be some kind of correspondence between the classes of the row and

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