ESTIMATION FOR A REGRESSION MODEL WITH AN UNKNOWN COVARIANCE MATRIX

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1. Summary and introduction

A linear regression model is considered under which the residual error vector is assumed to have a multivariate normal distribution with unknown covariance matrix Σ. To estimate Σ, it is assumed that the regression design can be given independent replications. This problem has been considered by Rao, who obtains a point estimator and suggests two classes of confidence regions for the vector β of regression parameters. In the present paper, we find the maximum likelihood estimators of β and of Σ, and derive their distributions. One of Rao's two classes of confidence regions for β had previously been inapplicable due to the lack of tables for upper tail values of the distribution of the pivotal quantity. These tables are now provided, and the performances of the two classes of confidence regions are compared in terms of their expected volumes.

In the classical linear regression model, the vector of observations $y = (y_1, y_2, \cdots, y_p)$ has the form

$$y = \beta X + \varepsilon, \tag{1.1}$$

where β: $1 \times q$ is an unknown vector of regression parameters, X is a known $q \times p$ matrix of rank $q \leqq p$, and ε has a p variate normal distribution with mean vector zero and covariance matrix $\Sigma = \sigma^2 I$. Since the simple structure of the covariance matrix may not be valid for some problems, extensions of the results of the classical model to models where Σ has a more general structure have been considered. Such attempts can be classified in the following hierarchy of complexity:

(i) Σ an arbitrary known matrix,

(ii) Σ known up to a scale factor σ^2,

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