

CREDIBILITY PROCEDURES

HANS BÜHLMANN

SWISS FEDERAL INSTITUTE OF TECHNOLOGY

1. Historical review

The concept of credibility is used by actuaries to estimate expected values (net premiums) from statistical data. The first papers on the subject were written by Whitney [8] and Perryman [7]. In the 1950's Arthur Bailey [1] in two special parametric cases gave a mathematical model from which the credibility procedures could be justified. Only in the last few years a nonparametric credibility theory was developed [3] which is now being further refined [4], [5]. The technique derived from the theory is becoming a major actuarial tool in non-life insurance.

As will be apparent from the formulation below, the method of estimation which actuaries mean when referring to credibility procedures is of quite general interest and can easily be transcribed to other fields of application where it may be used for forecasting. For reasons of intuitive appeal I shall, however, restrict the terminology to the actuarial application. The presentation here given follows in many respects that in [4] (in German).

2. The problem

For $i = 1, \dots, n; j = 1, \dots, N$, we consider random variables $X_{i,j}$, non-negative real numbers $P_{i,j}$, and maps $\rho_{i,j}$ from R^∞ to R^1 , where we think of j as indicating the risk (or risk group) within the collective of risks and i as indicating the period (say year) over which these risks (risk groups) can be observed. The above introduced abstract concepts have the following intuitive meaning:

$X_{i,j}$ is the observable risk performance (year i , risk j),

$P_{i,j}$ is the measure of exposure (year i , risk j), and

$\rho_{i,j}$ is the map assigning the risk performance $X_{i,j}$ to the doubly stochastic sequence of individual claims (year i , risk j). (We call $\rho_{i,j}$ the insurance conditions.)

Finally we introduce a parameter $\vartheta_{i,j}$ taking values in an abstract $\theta, i = 1, \dots, n; j = 1, \dots, N$, and we think of it as characterizing the "quality" of the risk j in the year i . Using all symbols just introduced we write