THE SPECTRAL ANALYSIS OF STATIONARY INTERVAL FUNCTIONS

DAVID R. BRILLINGER
UNIVERSITY OF CALIFORNIA, BERKELEY

1. Introduction and summary

We consider stationary, additive, interval functions $\mathbf{X}(\Delta)$. These are vector valued stochastic processes having real intervals $\Delta = (\alpha, \beta]$ as domain, having finite dimensional distributions invariant under time translation and satisfying

$$\mathbf{X}(\Delta_1 \cup \Delta_2) = \mathbf{X}(\Delta_1) + \mathbf{X}(\Delta_2), \tag{1.1}$$

for disjoint intervals Δ_1, Δ_2. Such processes are considered in some detail in Bochner [5]. Setting

$$\mathbf{X}(t) = \mathbf{X}(0, t], \tag{1.2}$$

$-\infty < t < \infty$, and in the reverse direction setting

$$\mathbf{X}(\alpha, \beta] = \mathbf{X}(\beta) - \mathbf{X}(\alpha), \tag{1.3}$$

we see that a consideration of stationary interval functions is equivalent with a consideration of processes $\mathbf{X}(t)$, $-\infty < t < \infty$, having stationary increments. These last are discussed in Yaglom [24] for example. Important examples of processes of the type under consideration are provided by the point processes. Here the components of $\mathbf{X}(\Delta)$ give the number of events of various sorts that occur in the interval Δ. A variety of properties and applications of point processes may be found in Cox and Lewis [11], Bartlett [4], and Srinivasan [21].

The paper is divided into various sections. In Section 2 we introduce a key assumption for the processes; specifically we require that all the moments of $\mathbf{X}(\Delta)$ exist and have particular integral representations. We are then able to define

$$f_{a_1, \cdots, a_k}(\lambda_1, \cdots, \lambda_k), \tag{1.4}$$

$-\infty < \lambda_j < \infty$, $a_1, \cdots, a_k = 1, \cdots, r$, the cumulant spectra of order k of the r vector valued $\mathbf{X}(\Delta)$. These turn out to be generalizations of the cumulant spectra of order k of a continuous time series discussed in Brillinger and Rosenblatt [9]. We then present a spectral representation for $\mathbf{X}(\Delta)$. This representation was introduced in Kolmogorov [17] for real valued processes with stationary increments. It takes the form

$$\mathbf{X}(0, t] = \int_{-\infty}^{\infty} \left[\frac{\exp\{i\lambda t\} - 1}{i\lambda}\right] d\mathbf{Z}_X(\lambda), \tag{1.5}$$