EFFICIENT ESTIMATION OF REGRESSION COEFFICIENTS IN TIME SERIES

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1. Introduction

This paper deals with estimating regression coefficients in the usual linear model. Let y be a T-component random vector with expected value

$$\mathscr{E}\mathbf{y} = \mathbf{Z}\boldsymbol{\beta}, \tag{1.1}$$

where $\mathbf{Z}$ is a $T \times p$ matrix of numbers and $\boldsymbol{\beta}$ is a p-component vector of parameters. (All vectors are column vectors.) For convenience we assume that the rank of $\mathbf{Z}$ is the number of columns, p. The covariance matrix of y is

$$\mathscr{C}(\mathbf{y}) = \mathscr{E}(\mathbf{y} - \mathbf{Z}\boldsymbol{\beta})(\mathbf{y} - \mathbf{Z}\boldsymbol{\beta})' = \boldsymbol{\Sigma}. \tag{1.2}$$

(Transposition of a vector or matrix is denoted by a prime.) Again for convenience we shall assume that $\boldsymbol{\Sigma}$ is positive definite. The problem is to estimate $\boldsymbol{\beta}$ on the basis of one observation on y when $\mathbf{Z}$ is known.

When $\boldsymbol{\Sigma}$ is known or is known to within a constant multiple, the Markov or Best Linear Unbiased Estimate (BLUE) is given by

$$\mathbf{b} = (\mathbf{Z}'\boldsymbol{\Sigma}^{-1}\mathbf{Z})^{-1}\mathbf{Z}'\boldsymbol{\Sigma}^{-1}\mathbf{y}. \tag{1.3}$$

The least squares estimate is given by

$$\mathbf{b}^* = (\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{y}. \tag{1.4}$$

The covariance matrix of the Markov estimate is

$$\mathscr{C}(\mathbf{b}) = (\mathbf{Z}'\boldsymbol{\Sigma}^{-1}\mathbf{Z})^{-1}. \tag{1.5}$$

The covariance matrix of the least squares estimate is

$$\mathscr{C}(\mathbf{b}^*) = (\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\boldsymbol{\Sigma}\mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}. \tag{1.6}$$

Both of the estimates are linear and unbiased.

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