ASYMPTOTIC DISTRIBUTION OF THE LOG LIKELIHOOD RATIO BASED ON RANKS IN THE TWO SAMPLE PROBLEM[1]

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1. Introduction

Let $X_1, X_2, \cdots, Y_1, Y_2, \cdots$, be independent random variables where the X (Y) have common *strictly increasing* and *continuous* distribution function F_1^* (F_2^*). Let $N = 2n$ and $W_{N,1} \leqq W_{N,2} \leqq \cdots \leqq W_{N,N}$ be a rearrangement of $X_1, X_2, \cdots, X_n, Y_1, Y_2, \cdots, Y_n$ in increasing order of magnitude, $n = 1, 2, \cdots$. Define

$$Z_{N,i} = \begin{cases} 0 & \text{if} \quad W_{N,i} \text{ is an } X, \\ 1 & \text{if} \quad W_{N,i} \text{ is a } Y, \end{cases} \tag{1.1}$$

$i = 1, \cdots, N$, and let $\mathbf{Z}_N = (Z_{N,1}, \cdots, Z_{N,N})$.

Let F_1 and F_2 be two arbitrary *strictly increasing continuous* distribution functions. Let

$$L_N = L_N(\mathbf{z}_N) = \frac{P(\mathbf{Z}_N = \mathbf{z}_N | F_1^* = F_1, F_2^* = F_2)}{P(\mathbf{Z}_N = \mathbf{z}_N | F_1^* = F_2^* = F_1)}. \tag{1.2}$$

Note that the denominator in the above does not depend on F_1 and is equal to $1/\binom{N}{n}$ and that the numerator is unchanged if F_1 and F_2 are replaced by $F_1 K^{-1}$ and $F_2 K^{-1}$, where K is a strictly increasing continuous distribution function. Let

$$\ell_N = \ell_N(\mathbf{z}_N) = \log L_N(\mathbf{z}_N). \tag{1.3}$$

From now on $P(E)$ will stand for the probability of the event E when the common distribution of X and Y are F_1^* and F_2^*, respectively. Our main aim is to prove the asymptotic normality of $\ell_N(\mathbf{Z}_N)$ under suitable conditions (see Theorems 5.1 and 5.2). The conditions imposed are $A1$, $A2$, $A3$, and B or $\bar{A}1$, $\bar{A}2$, and $\bar{B}$ (see

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