SPACINGS REVISITED

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1. Introduction

This paper surveys some of the developments which have appeared in the literature on spacings during the five years since the presentation of two papers [16] and [17]. The first of these, by Proschan and this author, deals with the asymptotic theory of a class of tests for Increasing Failure Rate (IFR) which are based on spacings, whereas the second paper surveys the substantial literature on spacings that had appeared prior to 1965. In the present article we also set out some open problems which still remain in the asymptotic theory of tests based on spacings.

The general area of limit theorems for dependent random variables is broad and complex, with no unifying methodology. For example, problems related to rank statistics, linear combinations of order statistics and stationary sequences all require different approaches. Limit theorems for spacings represent some of the more challenging problems involving dependent variables, and the various approaches used provide interesting comparisons.

2. Basic formulations

By spacings we refer to the gaps or distances between successive points on a line. Let $\{T_n: n \geqq 0\}$ be a sequence of random variables (r.v.) for which $T_0 \leqq T_1 \leqq T_2 \cdots$. The spacings are then the differences $\{T_i - T_{i-1}\}$. There is a basic ambiguity in the theory of spacings caused by the radically different assumptions which can be placed on the T process. These differences can clearly be seen for example between the three basic models outlined below.

Model I: order statistics. For fixed n, one is given independent random variables $X_1, X_2, \cdots, X_n$, with common distribution function (d.f.) F_X. One defines $T_1 \leqq T_2 \leqq \cdots \leqq T_n$ to be the order statistics of the sample and considers the spacings $D_i = T_i - T_{i-1}$. The range for i is $2 \leqq i \leqq n$ unless the support of F_X indicates that spacings D_1 and/or D_{n+1} may be defined. The usual situation under Model I is a hypothesis testing one in which the two hypotheses are

$$H_0: F_X \in \mathscr{F}_0, \qquad H_1: F_X \in \mathscr{F}_1. \tag{2.1}$$

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