CLASSES OF DISTRIBUTIONS APPLICABLE IN REPLACEMENT WITH RENEWAL THEORY IMPLICATIONS

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1. Introduction and summary

Age and block replacement policies are commonly used to diminish in-service failures. Unfortunately, for some items (say, those with decreasing failure rate), use of these policies may actually *increase* the number of in-service failures.

In this paper we determine the largest classes of life distributions for which age and block replacement diminishes, either stochastically or in expected value, the number of failures in service. We obtain bounds on survival probability, moment inequalities, and renewal quantity inequalities for distributions in these classes. We show that under certain reliability operations on components in a given class of life distributions (such as formation of systems, addition of life lengths, and mixtures of distributions), life distributions are obtained which remain within the class.

We consider items which perform a function that is to be continued over an indefinite period of time. To make this possible, an item which fails while in service is immediately replaced by a new item of the same kind.

Sometimes the interruption caused by an in-service failure is costly compared with the item replacement cost. If it is possible to make a "planned" replacement of an unfailed item, thus avoiding the high cost associated with a failure replacement, then planned replacements provide a practical means for avoiding reliance upon aged or worn items.

It has long been realized that for units with certain kinds of life distributions, planned replacements actually increase the frequency of failures. A goal of this paper is to identify the life distributions for which planned replacements are, or are not, beneficial.

We assume that the life lengths of all items to be placed in service are independent and have a common distribution F. Without further mention, we assume that $F(z) = 0$ for $z < 0$, and we denote the *survival function* by $\bar{F} \equiv 1 - F$.