# ISOTONIC TESTS FOR CONVEX ORDERINGS

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## 1. Introduction

The problem of testing the hypothesis that $F$ is a negative exponential distribution with unknown scale parameter against the alternative that $F$ has monotone increasing nonconstant failure rate ($F$ has Increasing Failure Rate, IFR) has been studied by a number of authors, some of whom are Proschan and Pyke [18], Nadler and Eilbott [17], Barlow [1], Bickel and Doksum [7], and Bickel [6]. Bickel and Doksum show that the test proposed by Proschan and Pyke is asymptotically inadmissible. They then take an essentially parametric approach to the problem. In particular they obtain the studentized asymptotically most powerful linear spacings tests for selected parametric families of distributions which are IFR when the parameter $\theta > 0$ and exponential when $\theta = 0$. Bickel [6] proves that these tests are actually asymptotically equivalent to the level $\alpha$ tests which are most powerful among all tests which are similar and level $\alpha$ (for the associated parametric problems).

Since the problem is essentially nonparametric, we take a nonparametric approach similar to the one taken by Chapman [10] and Doksum [12] in studying the problem of testing for goodness of fit to a specified distribution against stochastically ordered alternatives. In addition, we consider a more general class of problems which includes the problem of testing for monotone failure rate. The setup is similar to that in Barlow and van Zwet [2].

Let $\mathcal{F}$ be the class of absolutely continuous distribution functions $F$ such that $F(0) = 0$ with positive and right (or left) continuous density $f$ on the interval where $0 < F < 1$. It follows that the inverse function $F^{-1}$ is uniquely defined on $(0, 1)$. We take $F^{-1}(1)$ to be equal to the right endpoint of the support of $F$ (possibly $+\infty$) and define $F^{-1}(0) = 0$. For $F, G \in \mathcal{F}$ we say that $F$ is c-*ordered* (convex ordered) with respect to $G (F \underset{c}{\leq} G)$ if and only if $G^{-1}F$ is convex on the

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