EFFICIENCY ROBUSTNESS OF ESTIMATORS

PAUL SWITZER
STANFORD UNIVERSITY

1. Introduction

This introduction sets out some general available procedures to get large sample efficient estimates of a location parameter when the governing distribution f is well specified. The next section reviews some attempts to relax the f specification. The third section discusses how one chooses from a given repertoire of competing estimators. It is there advocated that their respective estimated standard errors be used to govern the choice and two methods are presented for estimating standard errors nonparametrically for this purpose. Some Monte Carlo comparisons are presented in Section 4 using sample sizes of 30, 60, and 120 together with a short tail and long tail f. The possibility of using sample determined weightings of selected estimators is also briefly explored.

Our discussion of efficiency robustness is set in the context of estimating a location parameter θ, say the center of a symmetric distribution on the real line. We use $\hat{\theta}$ to generically denote a translation invariant estimator of θ. If f is the density function of the distribution and is sufficiently well specified, then there are various general methods available to obtain large sample efficient estimators. Some of these are:

(A) the maximum likelihood estimator, that is, the value of θ which maximizes $L(\theta) = \Pi_1^N f(X_i - \theta)$ where $X_1, X_2, \cdots, X_N$ are a sample of size N from f. For example, if f is Laplace then $\hat{\theta}$ is the sample median; if f is normal then $\hat{\theta}$ is the sample mean; if f is logistic then the MLE is not easily obtained.

(B) the Pitman estimator, namely, $\hat{\theta} = \int \theta L(\theta) d\theta / \int L(\theta) d\theta$, where $L(\theta)$ is the sample likelihood as above. For example, if f is normal then $\hat{\theta}$ is the sample mean; if f is uniform on an interval of fixed length then $\hat{\theta}$ is the midrange.

(C) the midpoint of a symmetric confidence interval for θ based on the locally most powerful rank test for the specified translation family f. The confidence probability is a fixed α, and the LMPRT depends on f through $J(u) = f'(F^{-1}(u))/f(F^{-1}(u))$ for $0 < u < 1$. For example, if f is Laplace then $\hat{\theta}$ is the average of a symmetric pair of the ordered X values; if f is logistic then $\hat{\theta}$ is the average of a symmetric pair of the ordered Walsh averages. Walsh averages are averages of pairs of X values. With $\alpha \to 0$ this is the method of Hodges and Lehmann [5].

(D) a specified weighted average of the ordered X values, namely, $\hat{\theta} =$